Project Proposal:
Pseudospectral methods for understanding boundary layer transition in simple flows with non-normal operators

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I: Background and Problem Statement

In even the most elementary problems in boundary layer studies, there is a large difference between the best experimental observations of transition to turbulence and the behavior expected from linear stability analysis using eigenvalues. That is, boundary layer transition is often observed for much lower Reynolds numbers than is expected according to the eigenvalue analysis, or - in the case of Couette flow - even when transition is not predicted at all regardless of Reynolds number. This is often referred to as "subcritical transition to turbulence."

This phenomenon, as discussed at length by Trefethen in [6] and [7], can be accounted for by including the effect of three-dimensional disturbances such as streamwise streaks and streamwise vortices which become amplified due to the non-normality[1] of the operator matrices in these flows. These 3-D perturbations to the mean flow are not considered in standard, 2-D, linearized eigenvalue analysis of the often-used Orr-Sommerfeld equation and other similar techniques, which account for only Tollmien-Schlichting wave disturbances.

Mathematically, Trefethen suggests, these 3-D disturbances can be incorporated into the stability analysis by examining the "pseudospectra" of the non-normal linear operator L. These are formed by examining the contours of $||(\lambda I - L)^{-1}|| \geq \epsilon^{-1}$ for various ϵ. Further detail is found in [6].

As an example of this analysis, see Trefethen's example of Couette flow pseudospectra from [7] in Figure 1 below.

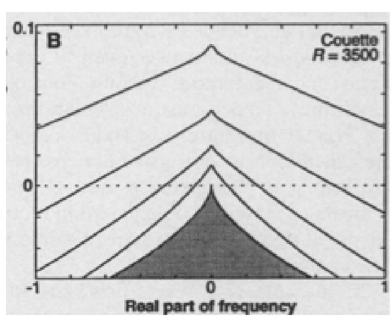

Figure 1: Pseudospectra for Couette Flow at $Re = 3500$

The shaded region denotes the spectrum of eigenvalues of the linear operator alone (note, never crossing into the unstable half-plane), and the set of distinct lines indicates the pseudospectrum, strongly protruding into the unstable half-plane, indicating the nonmodal effects will contribute significant growth behavior.

Other references to be used in this project: [1] [2] [3] [4] [5]

[1] That is, their eigenfunctions are not orthogonal to one another (though they are indeed linearly independent).

II: Problem Statement and Solution Path

For this course project, it is desired to complete the exercise of creating a custom numerical routine to examine the behavior of these non-normal operators (both the pseudospectra and resulting transient growth, specifically) for selected elementary flows[2]. The ultimate goal in this process will be to obtain a deeper first-hand knowledge of non-normal operators, and to fully understand how Trefethen's pseudospectum techiques can be implemented for practical use in hydrodynamic stability and other similar problems.

The analysis tool to be created will be entirely custom-designed for this project, and will be programmed and executed in the MATLAB software package.

References

[1] Jeffrey S. Baggett and Lloyd N. Trefethen. Low-dimensional models of subcritical transition to turbulence. *Physics of Fluids*, (96-236):11, 1996.

[2] William S. Saric, Ruben B. Carillo, and Mark S. Reibert. Nonlinear stability and transition in 3-d boundary layers. *Meccanica*, 33(5):469–487, Oct 1998.

[3] William S. Saric, Helen L. Reed, and Edward J. Kerschen. Boundary-layer receptivity to freestream disturbances. *Ann. Rev. Fluid Mechanics*, 34:291–319, Jan 2002.

[4] William S. Saric, Helen L. Reed, and Edward B. White. Stability and transition of three-dimensional boundary layers. *Ann. Rev. Fluid Mechanics*, 25:413–440, Jan 2003.

[5] Peter J. Schmid and Dan S. Henningson. *Stability and Transition in Shear Flows*. Springer, 2001.

[6] Lloyd N. Trefethen and Mark Embree. *Spectra and Pseudospectra*. Princeton University Press, 2005.

[7] Lloyd N. Trefethen, Anne E. Trefethen, Satish C. Reddy, and Tobin A. Driscoll. Hydrodynamic stability without eigenvalues. *Science*, 261(5121):578–584, Jul 1993.

[2] At the least, repeating Trefethen's Couette and Poiseuille Flow results, perhaps with a brief survey of others.